

Hong Kong Mathematics Olympiad (1998 – 99)

Final Event 1 (Individual)

香港數學競賽 (1998 – 99)

決賽項目 1 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

- (i) 若一個 P 邊的多邊形的角形成一算術級數，且最小和最大的角分別為 20° 及 160° ，求 P 之值。

If the angles of a P -sided polygon form an Arithmetic Progression and the smallest and the largest angles are 20° and 160° respectively, find the value of P .

$P =$

- (ii) 在 $\triangle ABC$ 中， $AB = 5$ ， $AC = 6$ 及 $BC = P$ 。若 $\frac{1}{Q} = \cos 2A$ ，求 Q 之值。

(提示： $\cos 2A = 2\cos^2 A - 1$)

In $\triangle ABC$, $AB = 5$, $AC = 6$ and $BC = P$. If $\frac{1}{Q} = \cos 2A$, find the value of Q .

(Hints: $\cos 2A = 2\cos^2 A - 1$)

$Q =$

- (iii) 若 $\log_2 Q + \log_4 Q + \log_8 Q = \frac{R}{2}$ ，求 R 之值。

If $\log_2 Q + \log_4 Q + \log_8 Q = \frac{R}{2}$, find the value of R .

$R =$

- (iv) 若兩數 R 和 $\frac{11}{S}$ 的積等於它們的和，求 S 之值。

If the product of the numbers R and $\frac{11}{S}$ is the same as their sum, find the value of S .

$S =$

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Final Event 2 (Individual)

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決賽項目 2 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

- (i) 若 x, y, z 為非零實數使得 $\frac{x+y-z}{z} = \frac{x-y+z}{y} = \frac{-x+y+z}{x}$ ，且 $a = \frac{(x+y)(y+z)(z+x)}{xyz}$ ，求 a 之值。

If x, y, z are non-zero real numbers such that

$$\frac{x+y-z}{z} = \frac{x-y+z}{y} = \frac{-x+y+z}{x} \text{ and } a = \frac{(x+y)(y+z)(z+x)}{xyz}, \text{ find the}$$

value of a .

$a =$

- (ii) 設 u 和 t 為正整數使得 $u+t+ut=4a+2$ 。若 $b=u+t$ ，求 b 之值。

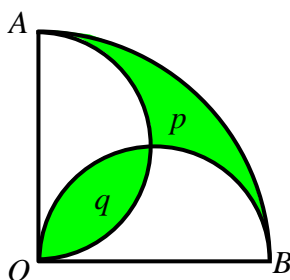
Let u and t be positive integers such that $u+t+ut=4a+2$. If $b=u+t$, find the value of b .

$b =$

- (iii) 在圖一， OAB 為四分之一圓，且以 OA 、 OB 為直徑繪出兩個半圓。若 p 、 q 代表斜線部份之面積，其中 $p=(b-9) \text{ cm}^2$ 及 $q=c \text{ cm}^2$ ，求 c 之值。

In Figure 1, OAB is a quadrant of a circle and semi-circles are drawn on OA and OB . If p, q denotes the areas of the shaded regions, where $p=(b-9) \text{ cm}^2$ and $q=c \text{ cm}^2$, find the value of c .

$c =$



圖一

Figure 1

- (iv) 設 $f_0(x) = \frac{1}{c-x}$, 且 $f_n(x) = f_0(f_{n-1}(x))$, $n = 1, 2, 3, \dots$ 。 若 $f_{2000}(2000) = d$, 求 d 之值。

Let $f_0(x) = \frac{1}{c-x}$, and $f_n(x) = f_0(f_{n-1}(x))$, $n = 1, 2, 3, \dots$. If $f_{2000}(2000) = d$, find the value of d .

$d =$

Hong Kong Mathematics Olympiad (1998 – 99)

Final Event 3 (Individual)

香港數學競賽 (1998 – 99)

決賽項目 3 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

- (i) 對任意整數 m 及 n ， $m \otimes n$ 之定義如下： $m \otimes n = m^n + n^m$ 。若 $2 \otimes a = 100$ ，求 a 之值。

For all integers m and n , $m \otimes n$ is defined as: $m \otimes n = m^n + n^m$. If $2 \otimes a = 100$, find the value of a .

$a =$

- (ii) 若 $\sqrt[3]{13b+6a+1} - \sqrt[3]{13b-6a-1} = \sqrt[3]{2}$ ，其中 $b > 0$ ，求 b 之值。

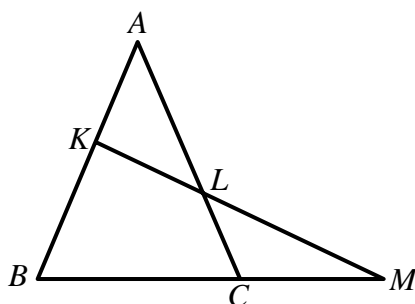
If $\sqrt[3]{13b+6a+1} - \sqrt[3]{13b-6a-1} = \sqrt[3]{2}$, where $b > 0$, find the value of b .

$b =$

- (iii) 在圖二， $AB = AC$ 和 $KL = LM$ 。若 $LC = b - 6$ cm 及 $KB = c$ cm，求 c 之值。

In Figure 2, $AB = AC$ and $KL = LM$. If $LC = b - 6$ cm and $KB = c$ cm, find the value of c .

$c =$



圖二

Figure 2

(iv) 數列 $\{a_n\}$ 的定義如下： $a_1 = c$, $a_{n+1} = a_n + 2n$ ($n \geq 1$)。若 $a_{100} = d$, 求 d 之值。

The sequence $\{a_n\}$ is defined as : $a_1 = c$, $a_{n+1} = a_n + 2n$ ($n \geq 1$). If $a_{100} = d$,
find the value of d .

$d =$

Hong Kong Mathematics Olympiad (1998 – 99)

Final Event 4 (Individual)

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決賽項目 4 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

- (i) 李先生今年 a 歲。若把李先生出生的月份與 a 相乘，其結果是 253，求 a 的值。

Mr. Lee is a years old. If the product of a and his month of birth is 253, find the value of a .

$a =$

- (ii) 李先生有糖 $a + b$ 粒。若平均分給 10 人，則餘下 5 粒。若平均分給 7 人，則欠 3 粒。求 b 之最小值。

Mr. Lee has $a + b$ sweets. If he divides them equally among 10 persons, 5 sweets will be remained. If he divides them equally among 7 persons, 3 more sweets are needed. Find the minimum value of b .

$b =$

- (iii) 設 c 為一正實數。若 $x^2 + 2\sqrt{c}x + b = 0$ 僅有一實數解，求 c 之值。

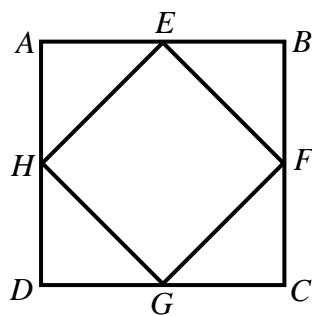
Let c be a positive real number. If $x^2 + 2\sqrt{c}x + b = 0$ has one real root only, find the value of c .

$c =$

- (iv) 在圖三，正方形 $ABCD$ 之面積為 d 。若 E 、 F 、 G 、 H 分別是 AB 、 BC 、 CD 、 DA 之中心點，且 $EF = c$ ，求 d 之值。

In Figure 3, the area of the square $ABCD$ is equal to d . If E , F , G , H are the mid-points of AB , BC , CD and DA respectively, and $EF = c$, find the value of d .

$d =$



圖三
Figure 3

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Final Event 5 (Individual)

香港數學競賽 (1998 – 99)

決賽項目 5 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

- (i) 若 $144^p = 10$ 、 $1728^q = 5$ 及 $a = 12^{2p-3q}$ ，求 a 之值。

If $144^p = 10$, $1728^q = 5$ and $a = 12^{2p-3q}$, find the value of a .

$a =$

- (ii) 若 $1 - \frac{4}{x} + \frac{4}{x^2} = 0$ 及 $\frac{a}{x} = b$ ，求 b 之值。

If $1 - \frac{4}{x} + \frac{4}{x^2} = 0$ and $\frac{a}{x} = b$, find the value of b .

$b =$

- (iii) 若方程式 $x^2 - bx + 1 = 0$ 有 c 個實數解，求 c 之值。

If the number of real roots of equation $x^2 - bx + 1 = 0$ is c , find the value of c .

$c =$

- (iv) 設 $f(1) = c + 1$ 及 $f(n) = (n-1)f(n-1)$ ，其中 $n > 1$ 。若 $d = f(4)$ ，求 d 之值。

Let $f(1) = c + 1$ and $f(n) = (n-1)f(n-1)$, where $n > 1$. If $d = f(4)$, find the value of d .

$d =$